Finite element investigation of thermal stresses in superconducting coils

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1 Motivation

The appearance of second generation high temperature superconducting materials (2G HTS) brought above the chance for this technology to gain ground in several fields. For example superconducting tapes are used widely in accelerators, motors, transformers and superconducting magnetic energy storage systems. However, accurate mechanical investigations are rarely been performed. This is the reason, why I started to work on this subject.

The main goal was to create a finite element model for the thermomechanical processes in the coil, based on Takematsu & Hu 2010 paper [1] and investigate the effects of changes like mesh fineness, element order, layer distribution, mechanical properties etc. During the research, changes were performed on the model in order to approach an actual coil.

The model has been implemented in COMSOL.

According to the goal described above, the 3rd chapter presents the finite element model of the layout presented in [1], but first I have performed a brief literature survey, which is summarized in the next (2nd) chapter. In the 4th and 5th chapter I made changes on the 2D model to match the reality as much as possible. Then in the 6th chapter a 3D model was created. The last chapter contains the summary of my work.
2 Literature survey

Before starting to work on a subject, it is important to make an overview on the past scientific researches and former results on that field, therefore the main papers written in the topic of “mechanical investigation of superconducting coils” were collected and they are summarised here.

One of the first papers in the topic was written by V. Arp in 1977: “Stresses in superconducting solenoids” [2]. They set up the elastic-continuum mechanics of cylindrically symmetric coils having anisotropic properties and apply it to superconducting magnets. Analytical equations are presented for winding tension, uniform and non-uniform temperature changes and magnetic forces. Since my work focuses on the thermal stresses in superconducting coils, section V. is relevant, where they present the differential equation expressed from the geometric, equilibrium and constitutive equations. It has to be noted, that the following neglections were used: $v_{rz} = 0; v_{gz} = 0; \alpha_z = 0$. The side-effects are also neglected, therefore the coil is considered to be infinite in width.

A similarly important paper was written by Takematsu and Hu in 2010 [1]. They performed voltage-current measurements on impregnated and not impregnated HTS coils (high temperature superconductivity) and they determined that the epoxy impregnated coil is more likely to degrade than the dry wound. They have also found the location of degradation, but for my work the most important part of this paper is Fig.4, where they present the radial stress distribution in the coil calculated based on the analytical equations from Arp’s paper [2]. In 3.1 a more detailed overview will be presented on this paper.

It is noticeable that in both [1] and [2] they calculated the radial stress in the superconducting coil. The reason for this is explained in [3] as follows:

“The mechanical strength of the YBCO tape under a tensile stress is high, at over 400 [MPa], but the delamination strength under a transverse tensile stress is low, at approximately 10 [MPa].”

This means that the reason of degradation is the transverse tensile stress, which delaminates the layers of the superconducting material. In this paper they also created 4 epoxy impregnated coils without degradation and they performed measurements on them. They compared the measured voltage-current curves with calculated ones.
The paper written by K. Ilin in 2015 [4] also contains important pieces of information, for example the material properties of the tape and its components. They also present measurement and FE results of stress and strain in a superconducting tape section under different mechanical loads. It has to be noted, that they did not investigate coils, just the tape itself.

Kozo Osamura and his team [5] carried out stress-strain measurements on SCS4050 superconducting material at different temperatures (5.7K, 77K, 298 K) and they evaluated the internal strain under tensile load at low temperatures down to 9.8K using neutron diffraction techniques. The strain dependence of the critical current was also determined.

In [6] S. Awaji and his team tested REBCO (REBa$_2$Cu$_3$O$_y$, RE: rare earth element) pancake coils under high magnetic field and large electromagnetic stress. They managed to perform stable operation without degradation and they carried out electric measurements at small and large hoop stresses.

Similarly, in [7] the strain induced by magnetic force on HTS double pancake coil was investigated. Measurements and analytical computations were performed and the results were compared. They also present analytically computed radial stress distribution in Fig.7, but during this calculation they neglected the thermal contraction, therefore these results are not relevant for my work.

In [8] the purpose was to establish a new thermal-stability criterion for YBCO tapes and to investigate the cause of degradation due to over-current. This new thermal-stability criterion was needed, because at that time (2010) the BSCCO tapes were replaced by YBCO tapes, as a new generation of superconducting materials. They carried out magneto-optical detection tests before and after the over-current test, which consisted the measuring of I-V characteristic. They also performed numerical simulations.

The presentation [9] of SuperPower Inc. (superconducting material manufacturer) from Applied Superconductivity Conference in 2010 presents different methods of testing the c-axis tensile straight of 2G HTS. This is the critical stress, which cause delamination, thus degradation of the tape. The results of these tests are also published. There are great differences between the presented results, but generally these critical stresses (50 [MPa]) are significantly larger than the 10 [MPa] from [1] and [3]. They investigated the location of degradation too.

The main conclusion of this overview is that the critical stress in a HTS coil is the radial stress, therefore it will be investigated in the followings using finite element method. The limit of this stress is not clearly determined in the literature, but for safety reasons, I will compare the results to the 10 [MPa], which is widely used in the literature. However the source of this 10[MPa] limit is not specified in the papers.
According to our theory, this 10 [MPa] can be a result of tests performed on coils (not only on the tape, like in the case of [9]). They might examined the degraded coils and calculated the radial stress, where the degradation happened, although it is not specified in the literature.
3 Reproduction of Takematsu & Hu’s results

This chapter describes the finite element model constructed based on the layout described in the paper and presents the results. It was important to start from a solid basis because this way I could validate the results before any modification was performed on the model.

The first section expands the previous overview of the paper written by Takematsu, the second shows the model and the third presents the results.

3.1 Overview on Takematsu & Hu’s paper

The purpose of the paper is to investigate a double pancake coil wound with a commercial YBCO-coated conductor (SCS4050) manufactured by SuperPower Inc. The topics investigated in the paper are as follows:

- The effect of epoxy impregnation on the current–voltage characteristics of YBCO-circular coils.
- The microstructure of the degraded YBCO-coated conductor in the coil winding.

The process of the experiment according to [1]:

“A dry wound double pancake coil was energized at 77K until the electric field in the coil exceeded 1 μV/cm, i.e. 560 μV (since the length of the conductor was 5.6 m). It was then warmed up to room temperature. The sequence of testing at 77K and warming up to room temperature was repeated five times.

The YBCO coil was then immersed in epoxy resin for several hours and hardened in an oven at 338 K for 50 h. It was cooled to 77 K and energized until the electric field in the coil again exceeded 1 μV/cm, i.e. 560 μV. This thermal cycle was repeated five times.”

The results of the experiment can be seen in Fig. 1. The conclusion was that the use of epoxy decreases the critical current by 42% and the current where normal voltage appears by 82%.

They also examined the location of degradation and they discovered that the YBCO conductor is fractured at the interface between the buffer layer and the YBCO layer, or at the YBCO layer itself in the 6th layer.
From the aspect of my modelling, the most important part of the paper is the calculated radial stress distribution in the epoxy impregnated double pancake coil due to cool down from room temperature to 77 K. This calculation was performed according to the analytical equations presented in [2]. The result can be seen in Fig. 2.

**Fig. 2:** Radial stress distribution in the coil winding due to cool down from room temperature to 77 K from [1].

This is the expected result from my finite element simulation.
3.2 Modelling

In this section, the information required for the numerical reproduction of the experiment is collected. It should be noted that there are several stress sources in a superconducting coil e.g. manufacturing, Lorentz forces, winding and contraction. In this work only thermal stresses are taken into consideration, the others are neglected.

3.2.1 Geometry

The original description of the coil according to [1]:

“The double pancake coil was wound with YBCO coated conductor (SuperPower Inc. SCS4050) around a 3 mm thick, fiber reinforced plastic (FRP) coil form without winding tension. The YBCO-coated conductor is 4.1 mm in width and 0.1 mm in thickness; a 35 µm thick Kapton tape is adhered on an outer side of the conductor. The YBCO coil winding is 30 mm in inner diameter, 38 mm in outer diameter and 8.8 mm in length. The number of turns is 27 X 2 and the length of the YBCO-coated conductor is 5.6 m.”

Based on this description, the model geometry for a single pancake coil is shown in Fig. 3.

![Fig. 3: The geometry of the finite element model based on the description in the paper.](image-url)
As Fig. 3 shows, the model contains only the half of the total width of the coil. This method can be used, because the geometry and all boundary conditions are symmetric. At first I use 2D axisymmetric finite element model to reduce computing time. Please note that this approach replaces the single tape helix with concentric circles. We can assume, that this simplification does not significantly influence the results since it is a commonly used approach.

The dimensions of the geometry can be found in Table 1.

<table>
<thead>
<tr>
<th>Name</th>
<th>Dimension [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>halfwidth</td>
<td>2</td>
</tr>
<tr>
<td>thickcoilform</td>
<td>3</td>
</tr>
<tr>
<td>Rcoilform</td>
<td>12</td>
</tr>
<tr>
<td>tapethick</td>
<td>0.1</td>
</tr>
<tr>
<td>kaptonthick</td>
<td>0.35</td>
</tr>
<tr>
<td>layerthick</td>
<td>0.148</td>
</tr>
</tbody>
</table>

*Table 1: Dimensions used for the finite element model*

### 3.2.2 Material properties

The source of the material properties is also the mentioned paper [1].

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus [GPa]</th>
<th>Relative thermal contraction [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRP length</td>
<td>18</td>
<td>-0.00211</td>
</tr>
<tr>
<td>FRP cross</td>
<td>14</td>
<td>-0.00638</td>
</tr>
<tr>
<td>Tape</td>
<td>200</td>
<td>-0.00211</td>
</tr>
<tr>
<td>Kapton</td>
<td>6</td>
<td>-0.0102</td>
</tr>
<tr>
<td>Resin</td>
<td>6</td>
<td>-0.0102</td>
</tr>
</tbody>
</table>

*Table 2: Material properties used for the finite element model*

Please note that in Takematsu &Hu’s paper the material properties of resin were not mentioned, therefore we assumed that its properties are the same as the Kapton’s, which is certainly the same magnitude. Also note, that for the FRP we used orthotropic linear material properties therefore the cross and tangential (hoop) directions were differentiated.
3.2.3 Modelling information

To reproduce the analytical radial stress distribution (Fig. 1), mentioned in the paper [1] and deduced in [2] the following approximations were applied on the FEM model:

- The width of the tape is approaching to infinite. As my studies confirmed in Fig. 4, 40 [mm] in width is large enough to reproduce the result in the middle of the tape, since 80 [mm] does not give noticeably different result.
- The thermal expansion is neglected along the axial direction ($\alpha_z = 0$).
- Each of the layers, and the coil form are connected elastically to each other.

With the approximations mentioned above, I managed to reproduce the analytical results. (“Coil form attached (wide)” in Fig. 5).

In the further investigation the model parameters should approach the actual coil, therefore the following changes were executed on the FEM model:

- The width of the tape was reduced to the actual size: 4 [mm].
- The layers were still connected elastically to each other, but they could be displaced at the coil form in axial direction.

The results computed after the first change (“Coil form attached”), and both changes (“Coil form not attached”) are also shown in Fig. 5, but first Fig. 4 confirms that 40 [mm] in width is enough to model the “infinite-width”.

Fig. 4: Confirming that 40[mm] in width is large enough to model the “infinite-width”
3.3 Results and comparison

The analytical results showed in [1] are compared with the FEM results in this chapter. The effect of mesh fineness, and element order is also examined.

3.3.1 Comparison of analytical and FEM results

The results of the original model (“infinite” width), and after the two changes (actual width and coil form displacement in axial direction) can be seen in Fig. 5 compared with the analytical result from [1].

As is can be seen in Fig. 5, I managed to reproduce the analytical results (red X) with the “infinite” width finite element model. When the width of the tape was reduced to its actual size, the radial stress decreased. When the layers can move in the axial direction next to the coil form, but they are attached to each other, the results does not change significantly. This was expected, because the thermal expansion in the axial direction is neglected.

In the followings I was trying to approach to the real thermomechanical problem, therefore I continue with the latest model (purple in Fig. 5).

![Fig. 5: FEM results compared to analytically calculated results](image-url)
3.3.2 The influence of mesh size

I also examined how the different meshes influences the radial stress in the coil. In this case the model I used is the same as in 3.3.1:

- 4 [mm] width.
- The thermal expansion is neglected along the axial direction ($\alpha_z=0$).
- Each of the layers are connected elastically to each other, but they can be displaced in axial direction next to the coil.

The utilized meshes are summarized in Table 3.

<table>
<thead>
<tr>
<th>Name</th>
<th>Element size [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarser2</td>
<td>0.04-0.06</td>
</tr>
<tr>
<td>Normal</td>
<td>0.02-0.03</td>
</tr>
<tr>
<td>Finer2</td>
<td>0.01-0.015</td>
</tr>
<tr>
<td>Finer4</td>
<td>0.005-0.0075</td>
</tr>
</tbody>
</table>

*Table 3: Different meshes*

In this case I did not only focused on the middle of the tape, but I also examined the radial stress at the following places:

- 90% of the half-width
- 95% of the half-width
- 98% of the half-width
- 99% of the half-width

For all of these simulations I used quadratic interpolation in the elements.

The results in the middle of the tape, and at 95% of the half-width are shown in Fig. 6. The other graphs do not contain additional information.
After examining Fig. 6 it can be determined that in the middle of the tape, the mesh size does not significantly influence the results. As we approach to the edge, the differences are increasing. As it was expected, with the finer meshes the results became smoother.

We also investigated the influence of the change in the order of element interpolation. The results are presented in the followings.
3.3.3 The influence of element order

In COMSOL we have the opportunity to choose the order of element interpolation used during the calculations. We thought that it would be useful to know, if this change has any significant effect on the results.

The results were calculated with the following element orders: Linear, Quadratic and Cubic.

Naturally we expect that with the cubic elements the curves will be smoother, but we do not expect great differences in the values.

For this comparison Finer4 mesh was used, because that provides the most accurate results.

Fig. 7 shows the radial stress along the radial coordinate at the middle of the tape, and at 98 % of the half-width. The remaining graphs do not add new information.

![Fig. 7: The influence of element type in the middle of the tape and at 98% of the half-width. In the right column curve details are shown.](image-url)
After looking at Fig. 7 we can conclude that the use of linear elements makes the curves jagged. Since they have less degrees of freedom, the calculation is faster, but the result becomes less accurate. Comparing the quadratic and cubic elements we can notice that the curve of radial stress with cubic elements is smoother, but the difference between the results is not significant. Thus in the following it is justified to use quadratic elements.

### 3.3.4 Radial stress along axial direction

As it can be noticed in Fig. 8, the results near the edge of the coil (e.g. 99% of the half-width) are getting “chaotic”. I would like to know that in the axial direction, how far we can go from the middle of the tape, where the results are still believable.

![Fig. 8: Radial stress distribution in radial direction at different locations](image)

To examine this, first I found the radial coordinate where the radial stress has maximum (aside from the side of the tape). This was at $R=0.01612$ [m]. At this location the radial stress along the axial direction can be seen in Fig. 9.
After looking at the figures, we can say that the different meshes has effect on the numerical results at the side of the tape. As we expected, “Coarser2” is the first one, which divides from the other 3, at \( Z=0.01925 \) [m]. It is also noticeable, that the “Finer2” and “Finer4” curves stay close.

To make it more clear, in Fig. 10 the relative difference is shown in percentage (supposing that “Finer4” provides the accurate result).

---

**Fig. 9: Radial stress in the axial direction at \( R=0.01612 \) [m] radial coordinate. On the right image the curves are shown in detail.**

**Fig. 10: The relative difference along the axial coordinate, compared to Finer4 at \( R=0.01612 \) [m] radial coordinate**
From Fig. 10 we can conclude that with the finer meshes the results remain plausible until greater axial coordinates, but even with the “Coarser2” mesh, the result is accurate until the 96.25% of the half-width.

To sum up, in this chapter I managed to reproduce the results published in [1], and I made a few steps towards the actual coil. I also examined the effect of mesh size and element orders. In the end, I determined the axial coordinate, until we can believe our results at the edge of the tape.

In the next chapter I continue to approach the actual coil by not neglecting the thermal expansion in the axial direction.
4 Model with thermal expansion in axial direction

In this chapter, an improved model is presented, which takes thermal expansion in axial direction into consideration. It is important to know the error caused by this neglection in [1] and [2]. Different meshes, and element orders were also tested here.

4.1 Modelling

In this case the model I use is almost the same as used at the end of 3.2.3. The only difference is the appearance of the thermal expansion in the axial direction:

- 4 [mm] width.
- The thermal expansion is not neglected along the axial direction ($\alpha_z \neq 0$).
- Each of the layers are connected elastically to each other, but they can be displaced in axial direction next to the coil.

4.1.1 Geometry and material properties

The geometry I use here is the one described in 3.2.1.

The source of the material properties is still [1], so they can be found in Table 2. The thermal expansion in the axial direction is the same as in the other directions for most of the materials. The only exception is the FRP, which has orthotropic material properties, therefore in the hoop (tangential) direction the “FRP length” data was used from Table 2.

4.2 Results

Since the goal was to determine the difference caused by the neglection of the axial thermal expansion, first we need to compare the results. In both cases the same meshes were used (Table 3). The influence of mesh size, and element order was also examined.

4.2.1 Difference caused by axial thermal expansion

In order to be able to compare the radial stress results, in both cases “Finer4” mesh was used with quadratic element interpolation. We expect to have a noticeable difference, because there are significant differences between the thermal expansion coefficients, presented in Table 3. The result are shown in Fig. 11.
As we expected, a quite significant difference can be seen in Fig. 11. In the middle of the tape, and at 90% of the half-width, the radial stress increases with the appearance of the thermal expansion. As we approach to the side of the tape, the size of peaks and notches grows drastically compared to the case without axial thermal expansion. The rest of the figures does not contain additional information.

**Fig. 11:** Comparison of radial stress with- and without thermal expansion in axial direction at the middle of the tape and at 90% and 95% of the half-width
4.2.2 The influence of mesh size and element order

In this case I also wanted to know the influence of the mesh size and the element interpolation order on the results. I run simulations with “Finer4”, “Finer2” and “Normal” meshes from Table 4 combined with linear, quadratic and cubic elements. Thus I got dozens of graphs.

The main conclusion, after analysing the results, was, that the use of linear element is not efficient, because the results become really jagged. In general, we can say that with finer mesh, and better element the curves become smoother. Naturally this is what we expected, but as before, the use of quadratic elements is adequate. A few diagrams are shown in Fig. 12.

![Fig. 12: Comparison of radial stress with different meshes-cubic element at 95% of the half-width and with different elements-“Finer4” mesh at 98% of the half-width. In the right column curve details are shown](image-url)
As it was mentioned before, there was no significant difference between the cubic and quadratic element types. This is shown on Fig. 12, in the second row, with a Finer4 mesh at 98% of the half-width. It is also noticeable on the graphs in the first row, that the Finer4 and Finer2 meshes provide almost the same results.

An interesting phenomenon is shown in Fig. 13. At 90% of the half-width, with linear elements, a “chaotic” section was found on the radial stress curve (around R=0.189 [m]), independently from meshes. Its explanation can be, that the linear element is not really accurate, but then it should appear at other locations too. This chaotic section does not appear with other element orders.

![Fig. 13: A “chaotic” section in the Radial stress distribution with linear elements, at 90% of the half-width](image)

Aside from this phenomenon, we got the expected results with the mesh, and element type examination. In this chapter I managed to compare the radial stress along the radial coordinate with, and without thermal expansion in axial direction. The difference was quite significant.

As further approach to the actual layout, the next step is to divide the tape into its component. This can be found in the next chapter.
5 Dividing the tape into components

From the mechanical point of view, our model is already approached the actual coil, but the tape is still considered as a homogenous material. This is obviously far from reality, but in [1] they used this approach, that’s why we also started the FE modelling this way.

According to the manufacturer’s data (Fig. 14), the superconducting tape (SuperPower SCS4050) consists of:

- Copper
- Silver
- Hastelloy (a kind of steel)
- YBCO
- Buffer

It is clear that I need to implement this change in the model, but the problem that I encountered was that we did not know the material properties of these layers. In the next section the sources of the new material parameters are presented.
5.1 Material properties of the tape components

Since the material properties strongly depend on the temperature, for this modelling it is important to find low temperature material properties. Therefore the sources of these data are mainly papers written in this topic, because in material tables usually room-, or high temperature data is published.

5.1.1 The manufacturer’s material properties

SuperPower provided a presentation [10] with test data of different type of superconducting tapes.

After digitizing the graphs, I performed bilinear curve fitting (the method will be presented later) on every data set in order to get the following mechanical properties:

- Young’s modulus
- Yield stress
- Hardening modulus (for later, plastic simulation)

The provided graphs are shown in Fig. 15 and the results of the curve fitting are in Table 4 and Table 5.

![Fig. 15: Test data provided by SuperPower. On the left graph the tape is without Ag and Cu (only. Hastelloy substrate (50µm thick)+YBCO)](image-url)
### Properties on 77K

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hastelloy+YBCO</td>
<td>180512,24</td>
<td>5086,16</td>
<td>0,00672</td>
<td>1207,21</td>
</tr>
<tr>
<td>SF12100</td>
<td>183900,30</td>
<td>7792,19</td>
<td>0,00713</td>
<td>1318,34</td>
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<tr>
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<td>4904,32</td>
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<td>5347,05</td>
<td>0,00666</td>
<td>978,26</td>
</tr>
<tr>
<td>SCS12050-40</td>
<td>126823,68</td>
<td>4362,07</td>
<td>0,00601</td>
<td>780,03</td>
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<td>SCS12050-100</td>
<td>76623,74</td>
<td>3856,61</td>
<td>0,00803</td>
<td>634,36</td>
</tr>
</tbody>
</table>

**Table 4: The results of curve fitting on 77K**

### Properties on room temperature

<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>Hastelloy+YBCO</td>
<td>179436,46</td>
<td>4001,55</td>
<td>0,00537</td>
<td>958,96</td>
</tr>
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<td>SF12100</td>
<td>184756,16</td>
<td>6197,21</td>
<td>0,00563</td>
<td>1046,98</td>
</tr>
<tr>
<td>SF12050</td>
<td>172887,94</td>
<td>3935,16</td>
<td>0,00544</td>
<td>943,99</td>
</tr>
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<td>SCS12050-20</td>
<td>138660,19</td>
<td>3721,40</td>
<td>0,00564</td>
<td>777,26</td>
</tr>
<tr>
<td>SCS12050-40</td>
<td>106310,17</td>
<td>3516,10</td>
<td>0,00565</td>
<td>607,98</td>
</tr>
<tr>
<td>SCS12050-100</td>
<td>87077,62</td>
<td>2603,04</td>
<td>0,00560</td>
<td>509,27</td>
</tr>
</tbody>
</table>

**Table 5: The results of curve fitting on room temperature**

From this source we managed to get a few material properties of different superconducting tapes, but since we need the properties of the tape components, only the “Hastelloy+YBCO” row can be useful, therefore other sources are needed.
5.1.2 Properties from Ilin & Yagotintsev-2015

In [4] by Ilin & Yagotintsev, experiments and FEM modelling of stress–strain state in superconducting tape under tensile were contributed.

In Figure 17 of the paper there are experimental stress-strain curves measured by them. I executed the bilinear curve fitting on these curves too, in order to compare with their data shown in Table 3 of [4]. They were using the same, SuperPower SCS4050 tape material, and they made experiments on the pure Hastelloy too. Their graph can be seen in Fig. 16.

![Fig. 16: The experimental stress-strain results from [4]](image)

In Table 6 there is a comparison between their results shown in Table 3 of [4], and our, curve fitting results based on Fig. 16. The source of the difference is the subjective factor in our bilinear curve fitting, since I have done the fittings as follows:

After importing the points of the graphs into Matlab, I determined an interval at the first (steep) section (Fig. 17/a) of the measurement data and I used the built-in Matlab function to fit a linear curve on these points. Then I similarly defined another interval after the yield point (Fig. 17/b) for the second linear curve. The slope of the first line is the Young’s modulus while the slope of the second linear curve is the hardening modulus. The subjective factor of this process is the designation of the two intervals, therefore I have done a few iterative steps to make the best fitting.

Fig. 17 contains the steps and the result of a curve fitting performed on the 77K-Hastelloy stress-strain curve from [4].
5. Dividing the tape into components

The comparison of material properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus [MPa]</th>
<th>Hardening modulus [MPa]</th>
<th>Yield strain [%]</th>
<th>Yield stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ilin</td>
<td>Our</td>
<td>Ilin</td>
<td>Our</td>
</tr>
<tr>
<td>Tape 77K</td>
<td>-</td>
<td>146766,29</td>
<td>-</td>
<td>13536,07</td>
</tr>
<tr>
<td>Hastelloy 77K</td>
<td>228000</td>
<td>227473,81</td>
<td>-</td>
<td>17785,62</td>
</tr>
<tr>
<td>Copper 77K</td>
<td>98000</td>
<td>81612,80</td>
<td>-</td>
<td>8997,82</td>
</tr>
<tr>
<td>Tape RT</td>
<td>-</td>
<td>138907,45</td>
<td>-</td>
<td>7748,05</td>
</tr>
<tr>
<td>Hastelloy RT</td>
<td>223000</td>
<td>221952,34</td>
<td>-</td>
<td>9695,72</td>
</tr>
<tr>
<td>Copper RT</td>
<td>80000</td>
<td>138907,45</td>
<td>-</td>
<td>7748,05</td>
</tr>
</tbody>
</table>

Table 6: The material properties from [4]. In “Ilin” columns the data is from Table 3 of the paper, while in “Our” columns the data is the result of bilinear curve fit for Figure 17 of [4] (RT stands for Room Temperature)
After looking at Table 6, it can be noticed that there is a significant difference in the yield stress in the case of Copper. The source of this difference can be the interpretation of yield stress, because normally it is the stress, where the material begins to deform plastically. In our case this was the intersection of the fitted linear curves (elastic & plastic).

In this paper ([4]) there are material properties for the YBCO layer too (in Table 2), and for Hastelloy and Copper they also provide thermal expansion properties. These are summarized in Table 7.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>YBCO</td>
<td>157000</td>
<td>1,1\times 10^{-5}</td>
<td>-2,43\times 10^{-3}</td>
</tr>
<tr>
<td>Hastelloy 77K</td>
<td>228000</td>
<td>1,34\times 10^{-5}</td>
<td>-2,96\times 10^{-3}</td>
</tr>
<tr>
<td>Copper 77K</td>
<td>98000</td>
<td>1,77\times 10^{-5}</td>
<td>-3,91\times 10^{-3}</td>
</tr>
<tr>
<td>Hastelloy RT</td>
<td>223000</td>
<td>1,34\times 10^{-5}</td>
<td>-2,96\times 10^{-3}</td>
</tr>
<tr>
<td>Copper RT</td>
<td>80000</td>
<td>1,77\times 10^{-5}</td>
<td>-3,91\times 10^{-3}</td>
</tr>
</tbody>
</table>

*Table 7: Young’s modulus and thermal expansion properties from [4]*

The data collected above can be more useful, because in [4] there were properties for copper, YBCO and Hastelloy, and thermal expansion coefficients were also given.

I also found a stress-strain curve for SCS4050 superconducting tape in Kozo Osamura’s paper [5] (Fig. 2), but that was not useful for us, since I will divide the tape and I need the mechanical properties of the components, not the full tape.
5.1.3 The manufacturer’s thermal expansion data

We also had thermal expansion data collected by the manufacturer of the tape. The source of the Hastelloy’s properties is MatWeb. It is essential to compare this with the one from [4].

The comparison is shown on Table 8.

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal expansion coefficient [1/K]</th>
<th>Relative thermal contraction [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ilin</td>
<td>SPI</td>
</tr>
<tr>
<td>Hastelloy</td>
<td>1,34·10^{-5}</td>
<td>9,27·10^{-6}</td>
</tr>
<tr>
<td>Copper</td>
<td>1,77·10^{-5}</td>
<td>1,39·10^{-5}</td>
</tr>
</tbody>
</table>

*Table 8: The thermal expansion properties from [4]. In “Ilin” columns the data is from Table 3 of the paper, while in “SPI” stands for the manufacturer’s data. The relative thermal contraction (l_0/l) is evaluated with ΔT=221[K].*

As it can be seen on Table 8, there is a notable difference between the data of the two sources, therefore in the followings we will compare the radial stress result caused by this thermal expansion difference.

Further thermal expansion coefficients can be found in N. Mitchell’s paper [11]. For the modelling, the properties of copper is relevant: αCu=1,31·10^{-5}. This is not significantly different from SPI data, thus hereinafter we will use that.

Now, as I have collected all of the needed mechanical properties, I can continue the modelling process.
5.2 Modelling

In this chapter the goal is to create a FE model in which the layers of the tape are handled separately. In the first section I collected the material properties, which were needed. In this one I describe the changes in the model compared to the previous chapter.

5.2.1 Geometry

The main changes were performed on the geometry, since I divide the tape into its components. Based on [4] we also decided to neglect the silver layer for now, since it is really thin, and we supposed that it does not have a noticeable influence on the results. The YBCO layer was neglected too first, but it will appear in the model later, because we wanted to examine the radial stress inside the YBCO layer. Based on these, the geometry can be seen in Fig. 18.

![Layer geometry](image)

*Fig. 18: The layer geometry of the finite element model, after modifications.*

The new dimensions are collected in Table 9.

<table>
<thead>
<tr>
<th>Name</th>
<th>Dimension [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuThick</td>
<td>0.025</td>
</tr>
<tr>
<td>SubThick</td>
<td>0.050</td>
</tr>
<tr>
<td>tapethick</td>
<td>0.1</td>
</tr>
</tbody>
</table>

*Table 9: New dimensions used for the finite element model*
5.2.2 Material properties

The material properties of the tape materials were collected in the previous section. The properties of FRP, Kapton, and resin can be found in Table 2.

First, I will use the SPI thermal expansion properties, and the Young’s modulus will be taken from [4].

5.2.3 Modelling information

In this section the modelling conditions does not change compared to the previous one.

I still use 2D axisymmetric model, with the geometry, and materials presented above. Some information about the model:

- 4 [mm] width.
- The thermal expansion is not neglected along the axial direction ($\alpha_z \neq 0$).
- Each of the layers are connected elastically to each other, but they can be displaced in axial direction next to the coil.

The results are presented in the next section.
5.3 Results

In this section the results what I got after dividing the tape are presented and I compare them with the original model. Since more thermal expansion property sets were found, they will be compared too. During my work I examined different meshes and element orders, but as before, they do not have really significant influence. Naturally we experienced that the finer mesh adds up to smoother curves.

5.3.1 The influence of dividing the tape

Here, the results I got before and after dividing the tape, are compared. For both simulations the same “Finer4” mesh and quadratic elements were used. The only difference is the material property set I used. For the “not divided” model the source of it is [1]. For the divided model, the source of Young’s modulus is [4] and the thermal expansion properties are from SPI table.

In Fig. 19 the radial stress at the middle of the tape is shown.

![Fig. 19: The radial stress at the middle of the tape with the “divided” and “not divided” model.](image)
5. Dividing the Tape into Components

As it can be seen in Fig. 19, the radial stress decreases with the tape division. The reason of this phenomenon can be the division itself, but the material properties of the divided tape materials does not add up to the material property of the tape from [1].

It is also interesting to analyse the radial stress close to the edge of the tape. This is shown in Fig. 20.

![Graphs showing radial stress comparison](image)

**Fig. 20:** The radial stress for the “divided” and “not divided” tape at the 98% and 99% of the half-width. In the right column the curves are shown in detailed view.

After analysing the curves in Fig. 20 we can notice that at 98% of the half-width the radial stress of the divided tape has huge pitches and notches, however at 99%, its “amplitude” is minor to the not divided tape’s.
5.3.2 The influence of thermal expansion properties

On Table 8 of 5.1 two different thermal expansion property sources and data were presented. Here I compare the radial stress along the radial direction with these two different thermal expansion data sets.

This is the only difference between the models, the same “Finer4” mesh and quadratic elements were used. The curves are presented in Fig. 21.

![Fig. 21: The radial stress with the “Ilin” and “SPI” thermal expansion coefficients at the middle and at 90% and 98% of the half-width. On the fourth graph the curves are shown in detailed view for 98% of the half-width.](image-url)
5. DIVIDING THE TAPE INTO COMPONENTS

On Fig. 21 we can see that there is a surprisingly significant difference caused only by changing the thermal expansion coefficients. For example, at the middle of the tape, at $R=0.015 \,[\text{m}]$ radial coordinate, in one case the radial stress is tensile, while in the other case it is compressive.

From this analysis we can conclude that the thermal expansion properties do strongly influence the radial stress along the radial axis. This means that since we have several sources and data for this material property, we can not totally trust the exact values of these simulations.

5.3.3 The influence of plastic behaviour

The next step was to examine, if the appearance of plasticity in the FE model has noticeable difference. This is the reason, why the bilinear curve fitting was done for all those curves in 5.1.

Here, the plastic behaviour is added for the tape materials in the FE model. I use Hardening modulus and Yield stress from Table 6.

For this simulation still “Finer4” mesh, quadratic elements and SPI thermal expansion coefficients were used. The results are summarized in Fig. 22.

![Fig. 22: The influence of plastic behaviour on the radial stress at the middle of the tape and at 99% of the half-width.](image)

As Fig. 22 shows, there is no noticeable difference caused by the appearance of plasticity at the examined locations. This is not surprising, because the radial stress does not exceed the yield stress from Table 6. Naturally this does not mean that plasticity has no effect on the stresses in the coil, it only means that at the investigated locations the stress peaks and notches are not large enough to show the difference.
5.3.4 The appearance of YBCO layer

The model with the divided tape was already presented, but the YBCO layer was neglected from that. Now, I add it to the model, in order to see the radial stress in the YBCO, since it is important to stay below a specific value to avoid the degradation (delamination) as it is presented in [1].

The only changes were contributed to the geometry. A 1 [μm] thick YBCO layer was added between the Hastelloy and the outer copper layers, and the thickness of the Hastelloy layer was reduced by 1 [μm] in order to leave the “tapethick” unchanged.

The material properties of the YBCO layer are presented in 5.1.

For this simulation a different mesh was used, which is finer next to the YBCO layer, and next to the edge of the tape, therefore I do not present the comparison graphs, but the appearance of YBCO does not influence the radial stress noticeable, since its thickness is only 1% of the “tapethick”.

The interesting data here is the radial stress along the axial direction in the middle of the YBCO layers. This is presented on Fig. 23.

![Fig. 23: The radial stress along the axial coordinate at different layers. The right graph shows the detailed curves next to the edge of the tape](image)

On Fig. 23 we can see that in most of the layers the radial stress is quite similar, but the last one (27\textsuperscript{th}) has significantly different stress values. The 25\textsuperscript{th} layer is divided from the others too, but it joins at R=0,0185 [m].
In this chapter I managed to develop the model by dividing the tape and using plastic materials. I also compared different thermal expansion coefficients and investigated the radial stress in the YBCO layer.

In the following I will keep these changes, thus plastic tape materials, and SPI thermal expansion coefficients will be used, because with that, the shape of the radial stress curves meets with our previous results, and the result from [1].

I think I have already looked into this topic quite well, several models and properties were presented, therefore it is time to move on and create a 3D model.
6 Creating a 3 dimensional model

In this section our goal is to create a 3 dimensional model of the coil. The purpose of this is that later we might want to take winding stress into consideration and that is not possible with a 2D axisymmetric model.

The first step is to reproduce the results, I got from the 2 dimensional model, and then I can make changes too.

In the first chapter I will create a FE model, which reproduces the radial stress results of the previous section. The YBCO layer will be kept in the model, and I will use the 3D version of the mesh, which was created in 5.3.4.

6.1 Modelling

In the 3 dimensional model the main changes compared to the 2D model are the followings:

- **Geometry**: we need to create a small part of the full geometry. It will be presented in detail in 6.1.1
- **Mesh**: a new mesh is created for this model. This is based on the one I used in 5.3.4, and it will be presented in the followings.

6.1.1 Geometry

The behaviour of the coil can not be precisely modelled by cuboid geometry, therefore hexahedrons will be used to build the model. It mainly looks like a cuboid, but its tangential thickness is not constant. It is increasing as we approach to the outer layers on the coil. The angle is only 0.003 [rad] but this barely visible change is responsible for the accurate results.

I created a simple geometry to demonstrate the difference between cuboid and hexahedron geometry, and they are compared with the 2D axisymmetric model. The geometry contains two domains with different thermal expansion properties. The results of the 3 models are shown on Fig. 24. We can notice that the difference is not significant for these parameters, but the red line, which belongs to the cuboid geometry, is detached from the other two curves. This suggests that with a hexahedron geometry the model is closer to the axisymmetric one.
The created 3D coil geometry is shown in Fig. 25.
In the figures ‘x’ is the radial, ‘y’ is the axial and ‘z’ is the tangential coordinate.

The dimensions of the layers are the same as before, from Table 1 and Table 9.

The minimal thickness of this “coil slice” is 50 [μm], at the inner side of the FRP.

### 6.1.2 Material properties, and modelling information

In this model I use the same material properties that was used for the last 2D simulation (5.3.4) because our goal is to reproduce that result with this model. The only changes were contributed on the boundary conditions. The main features:

- The thermal expansion is not neglected along the axial direction ($\alpha_z \neq 0$).
- Each of the layers are connected elastically to each other, but they can be displaced in axial direction next to the coil.
- 4 [mm] width.

I still use only the half of the width, because the tape is symmetric, therefore I prescribed zero displacement there in the ‘y’ direction (as before). For this 3D model a zero displacement in ‘z’ direction for the $z=0$ plane was also prescribed.

In order to ensure tangential strains in the ‘z’ direction the following displacement was prescribed for all of the domains in ‘z’ direction:

$$ w = \frac{u}{X} \cdot Z $$  \hspace{1cm} (1)

where:
- $w$ [m] stands for the displacement in ‘z’ direction
- $u$ [m] stands for the displacement in ‘x’ direction
- $X$ and $Z$ [m] are the general, undeformed coordinates.

With these additional boundary conditions I managed to reproduce the radial stress results of the last 2D model.

### 6.1.3 Mesh

For this model I needed to create a new mesh, since it is in 3D, and the YBCO layer is added too. I considered Finer4 as a base of this new mesh, but we performed changes, to avoid the sharp changes in element size. The mesh is shown in Fig. 26.
As Fig. 26 shows, the element size is reduced next to the YBCO layer, and the edge of the tape. The properties of the mesh are shown in Table 10.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Method</th>
<th>Element size [μm]</th>
<th>Number of elements</th>
<th>Element ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Copper</td>
<td>size</td>
<td>4-8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hastelloy</td>
<td>distribution</td>
<td>-</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>YBCO</td>
<td>distribution</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Outer Copper</td>
<td>distribution</td>
<td>-</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Kapton</td>
<td>size</td>
<td>4-8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Resin</td>
<td>size</td>
<td>4-8</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 10: Properties of the elements of Finer41 mesh

The mesh properties are presented in Table 10 in the radial direction. In the axial direction I used “distribution” too (in the swept command) with 80 elements and ‘40’ element ratio. In the tangential direction I defined only one element, and I verified that with one element I get the same result, like with 3 or 5. Therefore with only one element, I managed to reduce the degrees of freedom.

So the mesh, which will be used in the followings, and which was used in 5.3.4, is now specified.
6.2 Results and comparison

In this section the results of the 2D and 3D model are compared, and then further investigations are performed on the 3D model.

6.2.1 2D axisymmetric and 3D model

Before I start to use the 3D model, I need to be sure, that it gives approximately the same results, than the validated 2D model. For this reason I compare the radial stress result of the two models.

In both cases I have the same layout, including the YBCO layer, and the same mesh, specified in 6.1.3.

The comparison is shown in Fig. 27.

We can notice there that at the middle of the tape and at 90% of the half-width there is a small difference between the curves. The 2D symmetric model produces a “saw tooth” like radial stress distribution, while on the curve of the 3D model we can notice the bounding surfaces of the different materials.

As we approach the edge of the tape this difference disappears, presumably because the range of the radial stress increases, and this small difference become invisible. We can suppose that this small difference is caused by the change of element type.

The curves for the middle of the tape, and for 90% and 98% of the half-width are presented on the next page.
Fig. 27: Radial stress comparison of the 2D axisymmetric and 3D model at the middle of the tape, and at 90% and 98% of the half-width. The curves are shown in details in the right column.
According to the results presented above, we can conclude that the 3D modelling was successful and I can continue with other investigations.

### 6.2.2 Mesh optimization

The next information that we might need in the future is how many elements are needed along the half-width. It is important to know, because if we can reduce the number of elements, the computing time will decrease too.

The following cases were examined:

<table>
<thead>
<tr>
<th>Number of element along the half-width</th>
<th>Element ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>60</td>
<td>55</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>120</td>
</tr>
</tbody>
</table>

*Table 11: Element distributions along the half-width*

The goal was to find the coarsest mesh giving approximately the same result as the finest. Therefore I made two, coarser element distribution along the radial axis. These are presented in Table 12.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Method</th>
<th>Element size [μm]</th>
<th>Number of elements</th>
<th>Element ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Normal</td>
<td>Finer</td>
<td>Normal</td>
</tr>
<tr>
<td>Inner Copper</td>
<td>size</td>
<td>20-30</td>
<td>10-15</td>
<td>-</td>
</tr>
<tr>
<td>Hastelloy</td>
<td>distribution</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>YBCO</td>
<td>distribution</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Outer Copper</td>
<td>distribution</td>
<td>-</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Kapton</td>
<td>size</td>
<td>20-30</td>
<td>10-15</td>
<td>-</td>
</tr>
<tr>
<td>Resin</td>
<td>size</td>
<td>20-30</td>
<td>10-15</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 12: Properties of the elements of the used meshes*
In Fig. 28 we can find the radial stress distributions with several meshes. In the axial direction there are two element distribution, “Normal” and “Finer” as it is shown in Table 12. Along the half-width there are six element distribution, presented in Table 11.

In the middle of the tape there was no significant difference between the results, but at 90% of the half-width it is clearly noticeable, that the “Finer” and “Normal” curves stay close to each other, except the “15 element” curves, which are separated from the others.

After analysing the curves, the conclusion is that the minimum number of elements along the axial direction is 20, furthermore “Normal” radial element distribution also provides acceptable results.
6.2.3 Global-local analysis (submodelling)

The global-local analysis (also known as submodeling) is a method, which provides the opportunity of examining specified details of a whole system without drastically increasing the computing time. The process of an analysis is explained by Carlos A. Felippa in “Introduction to Finite Element Methods”\(^1\) as follows:

“The whole system is first analysed as a global entity, discarding or passing over details deemed not to affect its overall behaviour. Local details are then analysed using the results of the global analysis as boundary conditions. The process can be continued into the analysis of further details of local models.”

I used this method to investigate the effect of using significantly finer mesh in a part of the whole model, therefore I created a submodel containing the last 8 turns of the coil, and its width is half of the half-width. The layers have the same dimensions, and materials as before. The properties of the utilized mesh are collected in Table 13

<table>
<thead>
<tr>
<th>Layer</th>
<th>Method</th>
<th>Element size [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Copper</td>
<td>size</td>
<td>1-2</td>
</tr>
<tr>
<td>Hastelloy</td>
<td>size</td>
<td>1-2</td>
</tr>
<tr>
<td>YBCO</td>
<td>size</td>
<td>1</td>
</tr>
<tr>
<td>Outer Copper</td>
<td>size</td>
<td>1-2</td>
</tr>
<tr>
<td>Kapton</td>
<td>size</td>
<td>1-2</td>
</tr>
<tr>
<td>Resin</td>
<td>size</td>
<td>1-2</td>
</tr>
</tbody>
</table>

*Table 13: Properties of the elements of the local mesh*

Along the axial direction there are 80 elements with an element ratio of 20. In the tangential direction I still use only one element.

For the global-local analysis first the global model needs to be solved. This is the same that I presented above in 6.2.2. For that I used “Finer” mesh with 80 elements.

---

\(^1\) The full book can be downloaded from the page of a FEM course at University of Colorado, Boulder: http://www.colorado.edu/engineering/CAS/courses.d/IFEM.d/Home.html
The next step is to solve the local model. For the internal boundaries I prescribed the displacement from the solution of the global model.

The results of the global-local simulation are shown on Fig. 29.

Fig. 29: The radial stress results of global-local simulation. The bottom image shows only the last 8 turns.

From Fig. 29 we can say that with the local model we managed to reproduce the global results, and due to the finer mesh the curves are even smoother.
6.2.4 Inserting the silver layer

The next step is to take the silver layer into consideration in the model. I neglected this layer according to [4], and because its thickness is insignificant, but at last we wanted to have a model that contains everything.

The layer thicknesses are as follows:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner copper</td>
<td>22.5</td>
</tr>
<tr>
<td>Inner silver</td>
<td>2</td>
</tr>
<tr>
<td>Hastelloy</td>
<td>50</td>
</tr>
<tr>
<td>YBCO</td>
<td>1</td>
</tr>
<tr>
<td>Outer silver</td>
<td>2</td>
</tr>
<tr>
<td>Outer copper</td>
<td>22.5</td>
</tr>
<tr>
<td>Kapton</td>
<td>35</td>
</tr>
<tr>
<td>Resin</td>
<td>13.15</td>
</tr>
</tbody>
</table>

*Table 14: Layer thicknesses*

I developed the previous global-local model, therefore the element sizes in the mesh are the same.

Most of the material properties were already needed, except the silver’s. Its thermal expansion coefficient is from HyperPhysics\(^2\). \(\alpha=18\times10^{-6}[1/K]\), therefore the thermal contraction is 0.003942 [1].

The remaining mechanical properties were provided by FETI (Furukawa Electric Institute of Technology). \(E=90\ [GPa] ; \nu=0.33 ; \sigma_Y=150\ [MPa] ; H=8.24\ [GPa]\), where \(E\): Young’s modulus, \(\nu\): Poisson ratio, \(\sigma_Y\): yield stress, \(H\): hardening modulus.

The radial stress results are presented in Fig. 30.

After analysing the curves we can conclude that the negligence of the silver layer did not cause a great difference. The local model with the finer mesh gives smoother results, as we expected, but the difference is barely noticeable.

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\(^2\) HyperPhysics by Georgia State University: http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
Fig. 30: The radial stress results of global-local simulation with the silver layer in the model. The bottom image shows only the last 8 turns.
7 Summary

To sum up, during my work I managed to create a FE model, which reproduces the radial stress results presented in [1] (Fig. 2). Then the goal was to approximate our model to the real superconducting tape, therefore I executed the following changes on our model:

- reducing the width to 4 [mm]
- taking thermal expansion along the axial direction into consideration ($\alpha_z \neq 0$)
- allowing axial displacement next to the coil form
- dividing the tape into components
- taking plasticity into consideration
- creating the three dimensional model.

Beside this process, I investigated the influence of mesh fineness and element types and the material properties from different sources have also been compared.

The main conclusions:

1. The thermal expansion properties largely influence the radial stress results in the coil, therefore without the exact values of these properties the results of the simulation can not be taken infallible.

2. The superconducting coil is not expected to degrade as results of thermal stresses caused by the thermal contraction, because the stresses in the coil does not reach the critical stress of 10 [MPa] defined in literature. (Although this 10 [MPa] is not the only value that can be found. The critical stress of 50 [MPa] is presented in a different source, but this might belong to the structural strength, and the tape loses the superconducting properties at lower stresses.)

3. As it was presented in 6.2.2 the number of elements in the mesh can be greatly reduced, having beneficial effect on the simulation time.

4. Neglecting the silver layer from the layout does not have a significant impact on the results. (Although according to the latest information, it totally surrounds the substrate and the ceramic layer, so this layer might have some impact.)
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References


